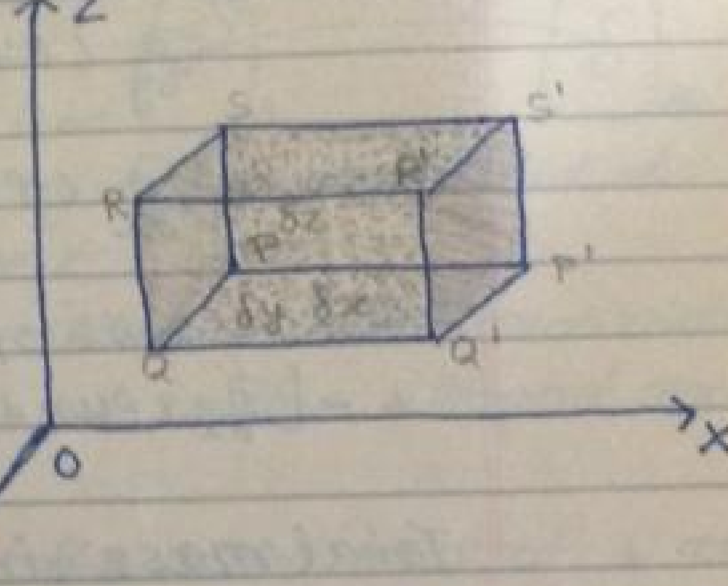
$\frac{29}{P-34}$ Equation of Continuity (cartesian cordinate)

PQRS   P'Q'R'S'
RR'S'S   QQ'PP'
RR'Q'Q   PP'S'S



Total accumulation = Express of the mass that flows in the Parallelopiped out the mass that flows out in time δt

Mass of the fluid that Passes in through the face PQRS

$= (\rho\, \delta y\, \delta z) u$ along x axis/u.m.

$= f(x, y, z)$

Mass of the fluid that Passes out through the face P'Q'R'S'

$= f(x+\delta x, y, z)$ along x axis Per unit mass

Excess of the mass that flows in through the face PQRS out the mass that flows out through the face P'Q'R'S'

$= f(x, y, z) - f(x+\delta x, y, z)$

$= f(x, y, z) - \{ f(x, y, z) + \delta x \frac{\partial}{\partial x} f(x, y, z) + \}$

$= -\delta x \frac{\partial}{\partial x} f(x, y, z)$

$= -\delta x \frac{\partial}{\partial x} (\rho u\, \delta y\, \delta z)$

$= -\frac{\partial}{\partial x} (\rho u)\, \delta x\, \delta y\, \delta z)$ along x-axis

2) Equation of stream line are

$$\frac{dx}{u} = \frac{dy}{v} = \frac{dz}{w}$$

$$\Rightarrow \frac{dx}{-\left(\frac{Ay}{x^2+y^2}\right)} = \frac{dy}{\left(\frac{Ax}{x^2+y^2}\right)} = \frac{dz}{0}$$

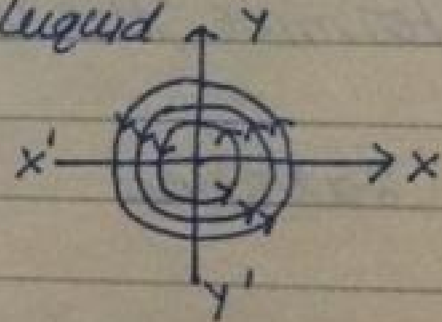
$$\Rightarrow -\frac{dx}{y} = \frac{dy}{x} \quad \text{and } z = \text{constant}$$

$$\Rightarrow x\,dx + y\,dy = 0 \;, \quad z = \text{constant}$$

$$x^2+y^2 = c^2 \;, \quad z = \text{constant}$$

It follows that the stream line are circles whose centre are on z-axis their Planes being Perpendicular to the axis.

Which is Possible liquid Motion.



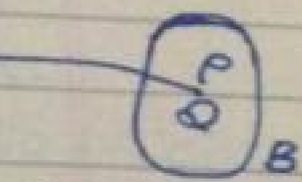
3) The motion is of Potential kind it follows that

$$\nabla \times q = 0$$

$$\nabla \times q = \begin{vmatrix} i & j & k \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ -\frac{Ay}{x^2+y^2} & \frac{Ax}{x^2+y^2} & 0 \end{vmatrix} = 0$$

Equation of Continuity (Lagrangian Method)

Let A be the region occupied by a fluid at the time $t=0$ and B be the region occupied by the same fluid at any instant of time t.



Consider (a, b, c) be the initial cordinates of a Particle P enclosed in this element and ρ_0 be its density.

Consider (x, y, z) be the cordinates of Q at any instant of time t and ρ be its density

Mass of the fluid element enclosing the Point P

$$= \rho_0 \, \delta a \, \delta b \, \delta c$$

Mass of the fluid element Q at any instant

$$= \rho \, \delta x \, \delta y \, \delta z$$

Total mass inside the region A = Total mass inside the region B

$$\iiint \rho_0 \, \delta a \, \delta b \, \delta c = \iiint \rho \, \delta x \, \delta y \, \delta z$$

$$\iiint \left[\rho_0 - \rho \frac{\partial(x\,y\,z)}{\partial(a\,b\,c)} \right] \delta a \, \delta b \, \delta c = 0$$

The region A is arbitrary hence

$$\boxed{\frac{\delta x \, \delta y \, \delta z}{\delta a \, \delta b \, \delta c} = \frac{\partial(x\,y\,z)}{\partial(a\,b\,c)} = \text{Jacobi}}$$

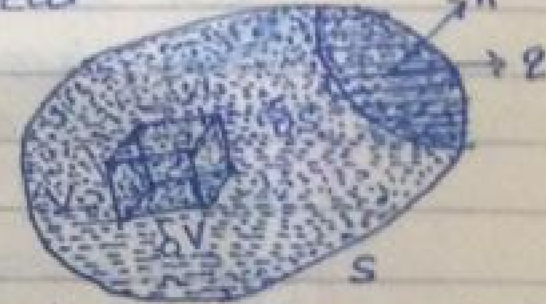
$$\rho_0 - \rho \frac{\partial(x\,y\,z)}{\partial(a\,b\,c)}$$

$$\rho_0 = \rho \frac{\partial(x\,y\,z)}{\partial(a\,b\,c)} \qquad \delta a \, \delta b \, \delta c \neq 0$$

$$\boxed{\rho_0 = \rho J}$$

Known as the eq^n of continuity in Lagrangian form.

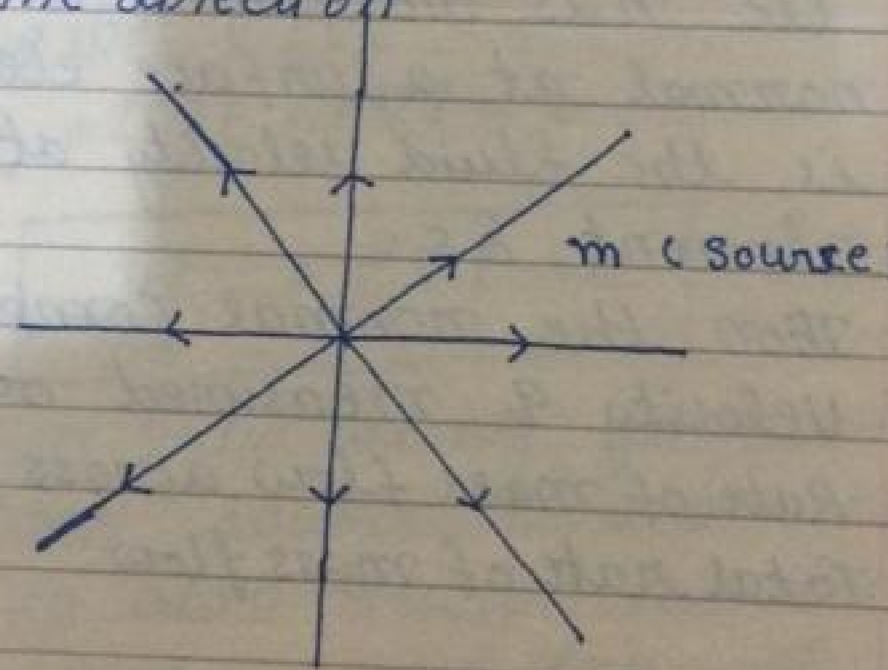
Equation of Continuity:-

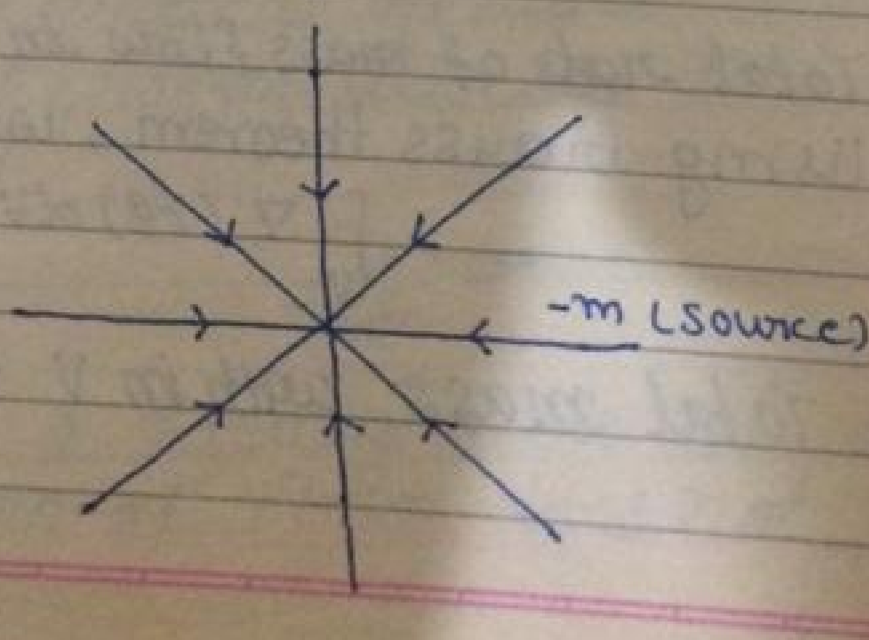


By continuity we mean Physical continuity
⇒ conservation of mass
the fluid always remain a continum
⇒ A continuously distributed matter. when a region of fluid contain neither sources nor sinks. then the amount of fluid within the region is conserved in accordance with the Principle of conservation of matter

In − out + source − sink = Accumulation

this is show conservation Principle
continouse certain from a Point symmetrycally in all the direction





$$= -\nabla \times (\nabla \phi) = 0$$

$$\boxed{\nabla \times q = 0}$$

Which is the necessary and sufficient condition for the motion to be irrotational

If $\nabla \times q \neq 0$ Rotational

—o—

Ex-23 / 52 $\quad q = A\dfrac{(xj - yi)}{x^2+y^2} \qquad A = \text{constant}$

$q = iu + jv$

To Prove

1) The motion is a Possible motion foran incompressible fluid.
2) To obtain the equation to streamlines
3) Motion is of Potential kind.
4) Determine the velocity Potential.

Sol^n 1) Satisfied $\quad u = -\dfrac{Ay}{x^2+y^2}$

$$v = \frac{Ax}{x^2+y^2}$$

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0$$

$$\Rightarrow -\frac{\partial}{\partial x}\left(\frac{Ay}{x^2+y^2}\right) + \frac{\partial}{\partial y}\left(\frac{Ax}{x^2+y^2}\right) = 0$$

$$\Rightarrow \frac{2Axy}{(x^2+y^2)^2} - \frac{2Axy}{(x^2+y^2)^2} = 0$$

$$0 = 0$$

Which is true Hence it is a Possible liquid motion